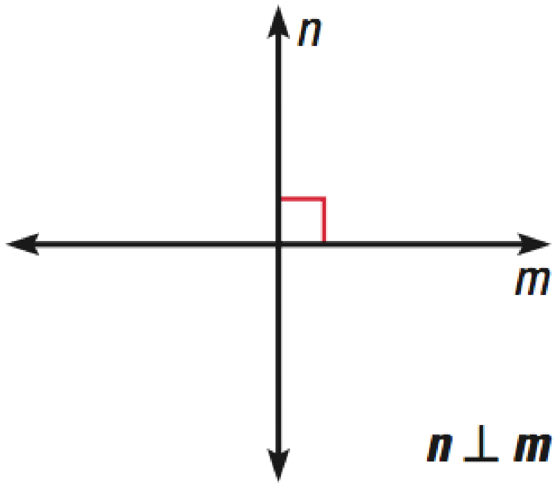
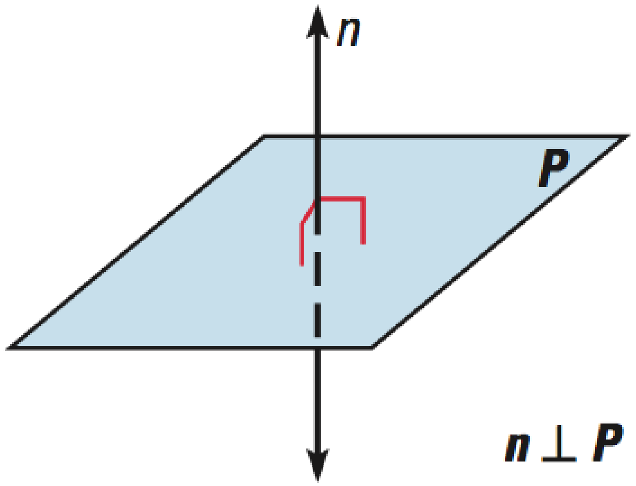
# Chapter 2
# Reasoning and Proof

# Section 2
# Definitions and Biconditional Statements

## GOAL 1: Recognizing and Using Definitions

Two lines are called __perpendicular__ if they intersect to form a right angle. A __a line perpendicular to a plane__ is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it. The symbol ⊥ is read as “is perpendicular to.”

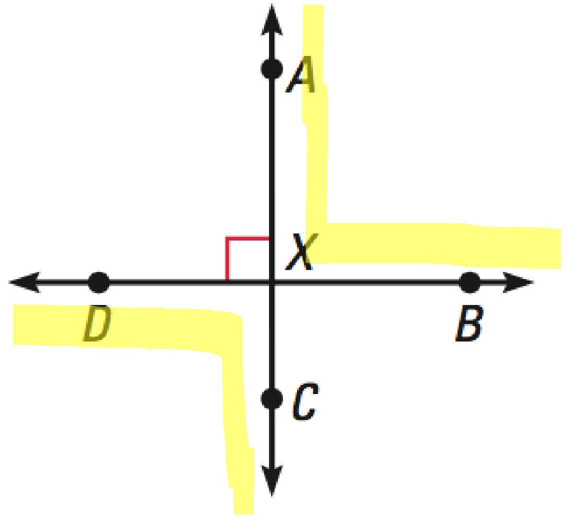


All definitions can be interpreted “forward” and “backward.” For instance, the definition of perpendicular lines means (1) if two lines are perpendicular, then they intersect to form a right angle, *and* (2) if two lines intersect to form a right angle, then they are perpendicular.

# Example 1: Using Definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.




a. Points D, X, and B are collinear.

Yes, all 3 points are on the same line

a. AC is perpendicular to DB.

Yes, they intersect to form a right angle

a. <AXB is adjacent to <CXD.

No, they don’t share a side (they are vertical angles)

GOAL 2: Using Biconditional Statements

Conditional statements are not always written in if-then form. Another common form of a conditional statement is only-if form. Here is an example.

$$\underbrace{\text{It is Saturday}}_{\text{Hypothesis}}\text{, only if } \underbrace{\text{I am working at the restaurant}}_{\text{Conclusion}}.$$

You can rewrite this conditional statement in if-them form as follows:

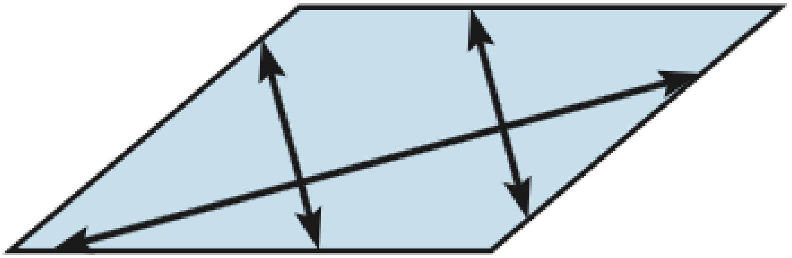
If **it is Saturday**, then **I am working at the restaurant.**

A ________biconditional statement__________ is a statement that contains the phrase "if and only if." Writing a biconditional statement is equivalent to writing a conditional statement *and* its converse.

Example 2: Rewriting a Biconditional Statement

The biconditional statement below can be rewritten as a conditional statement and its converse.

Three lines are coplanar if and only if they lie in the same plane.



Conditional Statement: (put “if” up front, replace “if & only if” with “then”)

If three lines are coplanar, then they lie in the same plane.

Converse:

If three lines lie in the same plane, then they are coplanar.

A biconditional statement can be either true or false. To be true, *both* the conditional statement and its converse must be true. This means that a true biconditional statement is true both “forward” and “backward.” All definitions can be written as true biconditional statements.

Example 3: Analyzing a Biconditional Statement

Consider the following statement: $x = 3$ if and only if $x^2 = 9$.

a. Is this a biconditional statement?

   Yes (it contains “if and only if”)

a. Is the statement true?

   Conditional: If x = 3, then x^2 = 9. TRUE

   Converse: If x^2 = 9, then x = 3. FALSE

   → Biconditional is false.

## Example 4: Writing a Biconditional Statement

Each of the following statements is true. Write the converse of each statement and decide whether the converse is true or false. If the converse is true, combine it with the original statement to form a true biconditional statement. If the converse is false, state a counterexample.

a. If two points lie in a plane, then the line containing them lies in the plane.

Converse:

If the line containing two points lies in the plane, then they are in the plane. TRUE

Biconditional:

Two points lie in a plane if and only if the line containing them lies in the plane.

Example 4: Writing a Biconditional Statement

Each of the following statements is true. Write the converse of each statement and decide whether the converse is true or false. If the converse is true, combine it with the original statement to form a true biconditional statement. If the converse is false, state a counterexample.

b. If a number ends in 0, then the number is divisible by 5.

Converse:

If the number is divisible by 5, then the number ends in 0. FALSE

Counterexample:

25 (any # ending in 5 would work)

Knowing how to use true biconditional statements is an important tool for reasoning in geometry. For instance, if you can write a true biconditional statement, then you can use the conditional statement or the converse to justify an argument.

Example 5: Writing a Postulate as a Biconditional

The second part of the Segment Addition Postulate is the converse of the first part. Combine the statements to form a true biconditional statement.

Postulate: If B lies between points A & C, then AB + BC = AC.

If AB + BC = AC, then B lies between points A & C.

Biconditional:

B lies between points A & C if and only if AB + BC = AC.